PART 1 - Machine Scored

1. An angle in standard position θ has reference angle of $30°$ with $sin\theta < 0$ and $tan\theta < 0$. A possible measure of θ, in radians, is:

A. $\frac{13\pi}{6}$ B. $-\frac{7\pi}{6}$ C. $-\frac{5\pi}{6}$ D. $\frac{23\pi}{6}$

Answers are on the back page *Full, worked out solutions can be found at www.rtdmath.com*

2. An angle in standard position θ has $cos\theta < 0$ and $cot\theta < 0$. The best estimate for the value of θ, in radians, is:

A. 1.05 B. 2.62 C. 3.92 D. 5.24

3. An angle in standard position θ has terminal arm that passes through a point $P(5, -4)$. The value of $sec\theta$ is approximately:

A. 1.28 B. -0.62 C. 0.78 D. -1.60

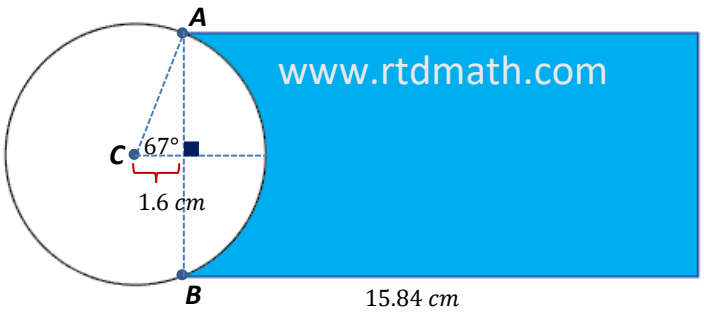
NR #1

An angle in standard position θ, where $0 \leq \theta \leq 2\pi$, has $sin\theta = -1/3$. Correct to the nearest tenth of a radian, the smallest possible value of θ is _____.

4. A point on the unit circle has coordinates $P(-\frac{5}{13}, m)$ and forms a principal angle in standard position, θ. If $tan\theta < 0$, then m is equal to:

A. $\frac{8}{13}$ B. $-\frac{8}{13}$ C. $\frac{12}{13}$ D. $-\frac{12}{13}$

Use the following information to answer question 5:

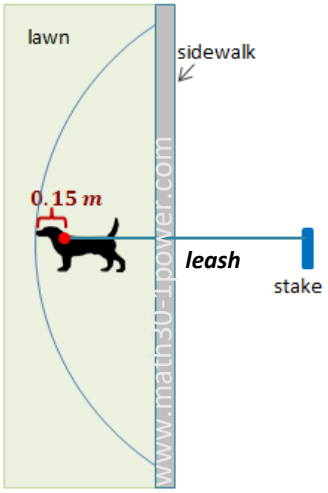


A rectangle with a base 15.84 cm in length intersects a circle with a centre **C**, at two points **A** and **B** as shown.

5. Correct to the nearest tenth, the **perimeter** of the shaded portion of the rectangle is:

A. 44.0 *cm* B. 46.8 *cm* C. 48.8 *cm* D. 49.5 *cm*

Use the following information to answer NR #2:

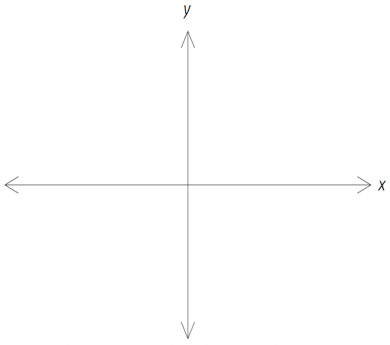


A dog has a leash tied to a stake in the ground such that she can reach a lawn on other side of a sidewalk as shown.

The dog is able to reach a horizontal distance of 0.15 m beyond the length of the leash, to form an arc on the lawn approximately 8.93 metres in length. The angle formed by lines connecting the stake and each end of the lawn arc is 110 degrees.

NR #2 The length of the leash, correct to the nearest tenth of a metre, is _____.

6. An angle in standard position θ passes through a point P as shown, where the x-coordinate is double the value of the y.



The value of $sec\theta$ can be written in the form $-\frac{\sqrt{a}}{b}$, where $a, b > 0$.
The value of $a + b$ is:

A. 3 B. 4 C. 6 D. 7

Use the following information to answer question 7:

A point on the unit circle has coordinates $P\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$, through which a terminal arm passes forming an angle θ. A second point has coordinates $Q\left(-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$ forming an angle β. Consider the following statements:

Statement 1: The smallest angle formed between the two terminal arms through P and Q is $\frac{5\pi}{12}$

Statement 2: One possible measure for θ is $-\frac{4\pi}{3}$

Statement 3: The value of $sin\theta$ is $\frac{\sqrt{3}}{2}$ and the value of $cos\beta$ is $-\frac{\sqrt{2}}{2}$

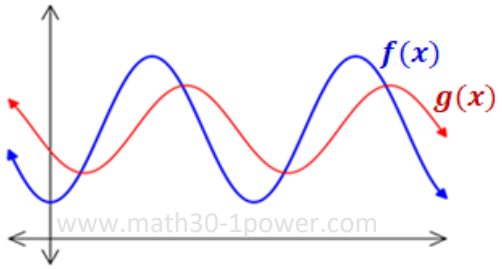
Statement 4: The value of $tan\theta$ is $-\sqrt{3}$ and the value of $cot\beta$ is 1

7. The number of true statements is:

A. 1 B. 2 C. 3 D. 4

8. The graph of a function $f(x)$ is shown, which can be expressed in the form $f(x) = a\ sin\ [b(x - c)] + d$.

The graph of $g(x)$ is obtained from $f(x)$ by changing the two parameters:



A. a and b

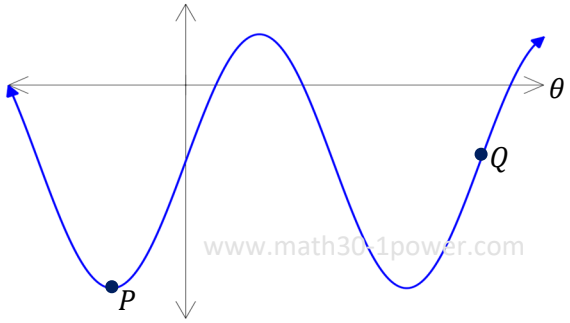
B. a and c

C. b and d

D. c and d

Use the following information to answer NR #3:

The graph shown models a sinusoidal function in the form $f(x) = asinx - d$, where $a > 0$ and $d > 0$. The point P is at a minimum.



Consider the following statements:

Statement 1: The angle represented by the point P is $-\pi/2$

Statement 2: The difference between the angle represented by the point Q and the angle represented by the point P is greater than 2π

Statement 3: The value of a is less than the value of d

Statement 4: The value of the y-intercept is $a - d$

Statement 5: If the function were transformed to $f(x) = asin\ (bx) + d$, where a and b are unchanged and $b < 1$, the horizontal distance between P and Q would increase.

At least two of the statements are true; that is two, three, four, or all five of the statements may be true.

NR #3 The true statements are statements number: *Enter in any order.*

Answers are on the back page *Full, worked out solutions can be found at* ***www.rtdmath.com***

9. A sinusoidal function has an equation $y = 5sin\ (4x + \pi)$. The value of the **period** and the **horizontal phase shift** are, respectively:

A. $\frac{\pi}{2}, \frac{\pi}{4}$

B. $\frac{\pi}{2}, \pi$

C. $4, \pi$

D. $4, \frac{\pi}{4}$

NR #4 A sinusoidal function has an f

The **period** of the resulting graph, correct to the nearest whole number, is a two-digit number ab (a *and* b *are the first two digits of your answer*)

The **maximum value** of the function, correct to the nearest tenth, is $c.d$
(c *and* d *are the last two digits of your answer*)

The values of a, b, c and d are:_____

10. The function $f(x) = tan\ (4x)$ has a domain, where $n \in I$, of:

A. $x \neq \frac{\pi}{4} + \frac{n\pi}{2}$

B. $x \neq \frac{\pi}{4} + \frac{n\pi}{4}$

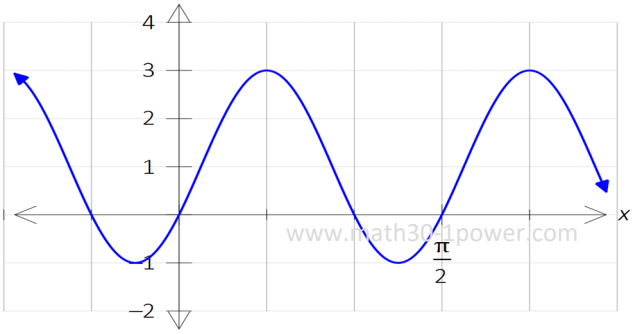
C. $x \neq \frac{\pi}{8} + \frac{n\pi}{2}$

D. $x \neq \frac{\pi}{8} + \frac{n\pi}{4}$

Use the following information to answer NR#5:

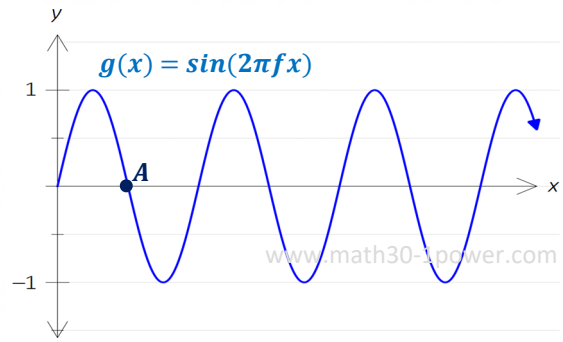
The partial graph of a cosine function with a max value of 3 and a min value of -1 is shown on the right.

The equation of the function can be written in the form $y = a\cos[b(x-c)] + d$, where a, b, and d are $\in I$.



NR #5 The value of b is _____ *first digit of your answer* and the value of d is _____. *second digit of your answer*

Use the following information to answer question 11:



A particular musical note can be modeled by a function $g(x) = sin(2\pi fx)$, where x is the time in seconds,

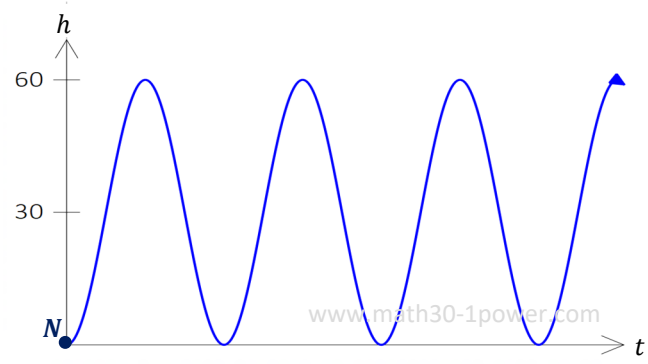
and f is the frequency of the note, measured in number of cycles per second, or hertz.

The graph of $g(x)$ is shown. It has an x-intercept after 0.00125 seconds, at a point A.

11. The frequency of the note modeled by the function above is:

A. 800π *Hz* B. 400π *Hz* C. 800 *Hz* D. 400 *Hz*

Use the following information to answer question 12:



The height of a nail caught in a tire rotating at a constant speed can be modeled by a sinusoidal function

$h = asin[b(t-c)] + d$

Where h is the height of a nail, in cm, after t seconds.

The graph shown models the height of a nail that starts at on the ground at a lowest position N at $t = 0$. The nail completes 20 rotations each minute.

12. The value of a and value of the phase shift c in the equation are, respectively:

A. 30 *cm, 0.75 sec* B. 30 *cm, 0.5 sec* C. 60 *cm, 0.75* D. 60 *cm, 0.5 sec*

13. Which of the following steps could lead to a correct solution of the equation $2cos^2\theta + 3cos\theta - 2 = 0$?

A. $cos\theta = \frac{1}{2}$ or $cos\theta = -2$ B. $cos\theta = \frac{1}{2}$ or $cos\theta = -1$ C. $cos\theta = \frac{-1}{2}$ or $cos\theta = 2$ D. $cos\theta = \frac{-1}{2}$ or $cos\theta = 1$

14. The solution, on $\{0 \le x \le 2\pi\}$, to $3csc^2\theta - 4 = 0$ is θ equal to:

A. $\frac{\pi}{3}, \frac{2\pi}{3}$ B. $\frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$ C. $\frac{\pi}{6}, \frac{5\pi}{6}$ D. $\frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

15. A general solution of the equation $sec^2x - secx - 2 = 0$, where $n \in I$ is:

A. $x = \frac{\pi}{3}n$ C. $x = \frac{\pi}{3}n + \frac{2\pi n}{3}$

B. $x = \frac{\pi}{3} + 2\pi n,\ x = \frac{5\pi}{3} + 2\pi n,\ x = \pi n$ D. $x = \frac{\pi}{3} + 2\pi n,\ x = \frac{5\pi}{3} + 2\pi n,\ x = 2\pi n$

16. The solution to the equation $log_2(tanx) + log_2(cosx) + 1 = 0$, where $\{0 \le x \le 2\pi\}$ is:

A. $x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{\pi}{2}$ B. $x = \frac{7\pi}{6}, \frac{11\pi}{6}$ C. $x = \frac{\pi}{6}, \frac{5\pi}{6}$ D. $x = \frac{7\pi}{6}, \frac{11\pi}{6}, \frac{\pi}{2}$

NR #6 The exact value of the trig ratio $cos\left(\frac{7\pi}{12}\right)$ can be determined to be an irrational expression in the form $\frac{\sqrt{a} - \sqrt{b}}{c}$ where a, b, c are positive integers.

The value of a is _____ *first digit*, the value of b is _____ *second digit* and the value of c is _____. *third digit*

17. A point $P(3, -5)$ lies on the terminal arm of an angle θ in standard position. The value of $\sin(\pi - \theta)$ is:

A. $\frac{3}{\sqrt{34}}$ B. $-\frac{3}{\sqrt{34}}$ C. $\frac{5}{\sqrt{34}}$ D. $-\frac{5}{\sqrt{34}}$

18. The non-permissible values of the expression $\frac{tanx}{1 + sinx}$ can be best written, where $n \in I$, as:

A. $x \ne \frac{3\pi}{2} + 2\pi n$ B. $x \ne \pi n,\ x \ne \frac{3\pi}{2} + 2\pi n$ C. $x \ne \frac{\pi}{2} + \pi n$ D. $x \ne \pi n,\ x \ne \frac{\pi}{2} + 2\pi n$

Use the following information to answer NR#7:

The simplified form of each of the following trigonometric expressions can be expressed using the indicated codes.

A $tanx + \frac{cosx}{1+sinx}$ **B** $\frac{cscx}{sinx} - \frac{cotx}{tanx}$

Use the following codes (in bold) to complete the sentence below:

1 1 **2** $cosx$ **3** $sinx$ **4** $secx$ **5** $cscx$ **6** $tanx$

NR #7 Expression **A** simplifies to _____ *first code* and expression **B** simplifies to _____ *second code*.

19. Given an angle θ in standard position where $cot\theta = -\frac{4}{5}$ and $sin\theta > 0$, the value of $sin\left(\frac{\pi}{3} + \theta\right)$ is:

A. $\frac{4\sqrt{3}-5}{2\sqrt{41}}$ B. $\frac{5-4\sqrt{3}}{2\sqrt{41}}$ C. $\frac{5\sqrt{3}-4}{2\sqrt{41}}$ D. $\frac{4-5\sqrt{3}}{2\sqrt{41}}$

A student is working on the left side of a proof for the identity $\frac{1+sinA-cos2A}{cosA+sin2A} = tanA$ when she makes a mistake. Her work is shown below:

Step 1 $\frac{1+sinA-cos^2A-sin^2A}{cosA+2sinAcosA}$

Step 2 $\frac{1-cos^2A+sinA-sin^2A}{cosA+2sinAcosA}$

Step 3 $\frac{sinA}{cosA+2sinAcosA}$

Step 4 $\frac{1}{3cosA}$

20. The first recorded mistake occurs in:

A. *Step 1* B. *Step 2* C. *Step 3* D. *Step 4*

Use the following information to answer NR#8:

An angle in standard position θ terminates in quadrant II, with $cos\theta = -4/5$.

NR #8 The expression $tan2\theta$ simplifies to $-\frac{a}{b}$, where a, b are positive integers, a can be expressed in two digits and b is one. The three digits representing the values of a and b are _____.

PART 2 - Written Response

Answers are on the back page *Full, worked out solutions can be found at* ***www.rtdmath.com***

Use the following information to answer WR#1:

An angle in standard position θ passes through a point $P(-5, 1)$ and a second angle in standard position β passes through a point $Q(-3, -4)$.

❖ *Written Response Question 1*

- Fully **sketch** each angle in the correct quadrant labeling all sides of the triangle, and **determine** the value of each angle, correct to the nearest degree. ***(3 marks)***

- **Determine** the exact value of $sin\,(\theta + \beta)$, written in the form $\frac{p}{q}$ ***(2 marks)***

❖ *Written Response Question 2*

- Using a trigonometric identity, **simplify** the equation $2sin^2x - cosx - 1 = 0$ to express in terms of one trig function, where the lead coefficient is positive. ***(2 marks)***

- **Algebraically solve** the resulting equation on $\{0 \leq x < 2\pi\}$, and state a general solution. ***(3 marks)***

❖ *Written Response Question 3*

- **Prove** the equation $\frac{cscx\,cosx}{tanx + cotx} = cos^2x$ is an identity using an algebraic approach. ***(3 marks)***

• **Determine** each of the possible non-permissible values, in radians. ***(2 marks)***

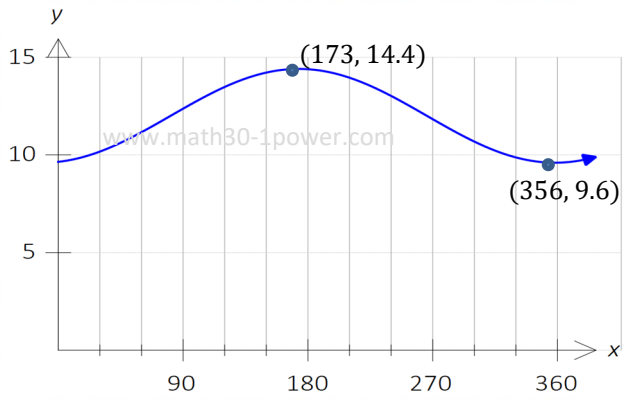
Use the following information to answer part #4

In San Diego, California the number of hours of daylight follows a sinusoidal pattern where the maximum hours of sunlight is 14.4 hours on day 173 (June 27th), and the minimum hours of sunlight is 9.6 hours on day 356 (Dec 22nd).

The function below is for a particular leap year of 366 days.

The hours of sunlight (H) can be modeled as a cosine function of day number (x):

$H = a\ cos\ [b(x - c)] + d$



❖ *Written Response Question 4*

• **Determine** the values of a, b, c, and d in the equation $H = a\ cos\ [b(x - c)] + d$ ***(3 marks)***

• The daily high temperature in San Diego can be modeled by the function $T = 5.1sin\ [0.524(d - 2.75)] + 23.9$, where T is the temperature in degrees Celsius, and m is the number of months from the start of the year.

Use a graphing approach to **determine** the approximate total number of months, correct to the nearest tenth, where the daily high temperature would be above $26℃$. ***(2 marks)***

• A function of similar form to the last bullet is constructed for Calgary Alberta, where the temperatures are much cooler. **Explain** which of the two parameters a, b, c, and d would be different, and how. **Justify** your reasoning.
(Note, on the actual diploma exam each WR question will have exactly two bullets)

Answers

For full, worked-out solutions (as well as other practice materials) visit **www.rtdmath.com**

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Multiple Choice

1. D **2.** B **3.** A **4.** C **5.** C **6.** D **7.** C **8.** B **9.** A **10.** D **11.** D

12. A **13.** A **14.** B **15.** C **16.** C **17.** D **18.** C **19.** B **20.** A

Numerical Response

1. 3.5 **2.** 4.5 **3.** 125 **4.** 5281 **5.** 41 **6.** 264 **7.** 41 **8.** 247

Written Response

1. *First bullet* $\theta = 169°\ \ \beta = 233°$ *Second bullet* $\frac{17}{5\sqrt{26}}$

2. *First bullet* $2cos^2x + cosx - 1 = 0$ *Second bullet* $x = \frac{\pi}{3}, \pi, \frac{5\pi}{3}$ (any order) $x = \frac{\pi}{3} + \frac{2\pi}{3}n\ \ n \in I$ (general sol.)

3. *First bullet* See full solutions on www.rtdmath.com *Second bullet* $x \neq \frac{\pi}{2}n\ ,\ n \in I$

4. *First bullet* $a = 2.4,\ b = \frac{\pi}{183},\ c = 173,\ d = 12$ *Second bullet* $\mathbf{4.4}$ total months above 26℃.

Third bullet $\boldsymbol{a}$ would be **higher**, as the range of Calgary temperatures (between min and max) would be greater

$\boldsymbol{d}$ would be **lower**, as the median temperature for Calgary (represented by d) would be lower

Also…. (not needed in your answer)

$\boldsymbol{b}$ would be **unchanged**, as the period for each city would be the same (12 months). Similarly, $\boldsymbol{c}$ would be essentially unchanged, as the number of months after which the min / max temperature occurs would be approximately the same as both cities are in the northern hemisphere.